E1-2013-113

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SIMULATION OF DEPENDENCE OF THE CROSS SECTION OF BEAM DEUTERONS FRAGMENTATION INTO CUMULATIVE PIONS AND PROTONS ON THE MASS OF THE TARGET NUCLEUS

Submitted to «Ядерная физика»

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Моделирование зависимости сечения фрагментации
пучковых дейтронов в кумулятивные пионы и протоны
от атомной массы ядра мишени

Рассматриваются механизмы, влияющие на рождение кумулятивных пионов и протонов в реакции фрагментации налетающих дейтронов в кумулятивные пионы и протоны, испущенные под нулевым углом. В работе показано, что периферическая зависимость сечения от атомной массы ядра мишени, наблюдавшаяся в эксперименте, может быть объяснена рассеянием на нуклонах ядра мишени без введения дополнительных параметров.

Работа выполнена в Лаборатории физики высоких энергий им. В. И. Векслера и А. М. Балдина ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 2013

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Simulation of Dependence of the Cross Section
of Beam Deuterons Fragmentation into Cumulative Pions and Protons
on the Mass of the Target Nucleus

We consider the mechanisms that affect the production of cumulative pions and protons in the fragmentation of incident deuterons into cumulative pions and protons emitted at zero angle. We argue that the peripheral dependence of the scattering cross section on the atomic mass of target nuclei, which was detected in experiments with medium and heavy nuclei, can be described by scattering on target nucleons without introducing additional parameters.

The investigation has been performed at the Veksler and Baldin Laboratory of High Energy Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 2013

INTRODUCTUION

Regularities of the production of cumulative particles have been studied for more than forty years. In addition to independent interest in the detailed study of interactions of hadrons and nuclei in a specific kinematic region, the study of reactions with the production of cumulative particles can provide information on the structure of nuclei at short internucleonic distances (see [1–5]). Note that the experimental data indicates a number of interesting and not well understood regularities. In particular, approximate scaling of the spectra of hadrons depending on the scaled cumulative variable, increased dependence on the atomic mass of the fragmenting nucleus, etc. A consistent model to describe all experimentally established regularities of the production of cumulative particle has not yet been suggested. In this work, we study the role of processes of rescattering of cumulative proton or pion, emitted at zero angle, on a nucleonic target in a process of the fragmentation of the incident deutrons.

In order to define what is meant by the term «cumulative particle», in this article we consider the inclusive reaction:

$$A_{\rm I} + A_{\rm II} \to c + X, \tag{1}$$

where at least one of the colliding particles, $A_{\rm I}$ or $A_{\rm II}$, is a nucleus. The produced particle c is called «cumulative» if the following two conditions are fulfilled:

1. The particle c was produced in the reaction (1) in the kinematic region, that is inaccessible in the collision of free nucleons with the same momentum per nucleon, as $A_{\rm I}$ and $A_{\rm II}$.

2. The particle c belongs to the fragmentation region of one of the colliding particles, i.e., one of the following conditions should be met:

$$|Y_{A_{\rm I}} - Y_c| \ll |Y_{A_{\rm II}} - Y_c| \tag{2}$$

or

$$|Y_{A_{\rm II}} - Y_c| \ll |Y_{A_{\rm I}} - Y_c|, \tag{3}$$

where Y_i — rapidity of the corresponding particle i.

The second condition shows that the colliding particles appear in this definition asymmetrically. We will call the particle (one of the colliding) whose rapidity is closer to the rapidity of a cumulative particle as a fragmenting particle. The fragmenting particle must be a nucleus. The other colliding particle on which fragmentation occurs, we call fragmented particle. Untill now, experiments with the production of cumulative particles have been carried out so that the particle is detected outside of the rapidity interval $[Y_{A_{\rm II}}; Y_{A_{\rm I}}]$. The cumulative particles are detected either inside the backward hemisphere (target fragments) or inside the forward one (beam fragments). In this case, the second condition is the requirement of sufficiently high energy of collision:

$$|Y_{A_{\rm II}} - Y_c| < |Y_{A_{\rm I}} - Y_c| = |Y_{A_{\rm II}} - Y_c| + |Y_{A_{\rm II}} - Y_{A_{\rm I}}|. \quad (4)$$

In addition to the features of cumulative particles listed above, another one, directly related to the topic of this work, should be noted. We are talking about the results of studying the dependence of the reaction cross section on the mass of the nucleus, on which the fragmentation occurs. The analysis of the experimental data of [6] and [7] shows that in the case of the fragmentation of the projectile nucleus into cumulative particles, the dependence of the cross section on the atomic weight of the target nucleus A_t is different for cumulative and noncumulative areas. So, if to use for medium and heavy nuclei ($A_t \geq 12$) a power dependence of the inclusive cross section of the production of cumulative protons and pions on the atomic mass of the target nucleus (nucleus on which fragmentation occurs)

$$E d\sigma / d\mathbf{p} \sim A_t^n, \quad (5)$$

then for a noncumulative area this dependence is close to $n \sim 2/3$ (surface), whereas for the cumulative area this dependence is weaker $n \sim 1/3$ (peripheral). The corresponding experimental data is shown in Fig. 1 for protons and pions. Close relations were obtained in [8] and [9]. In the works [10] and [11], the conclusion about the peripheral nature of the reaction of fragmentation of the deuteron and carbon into cumulative pions at zero angle was made from the analysis of the concomitant multiplicity. Note that this change of the dependence on the

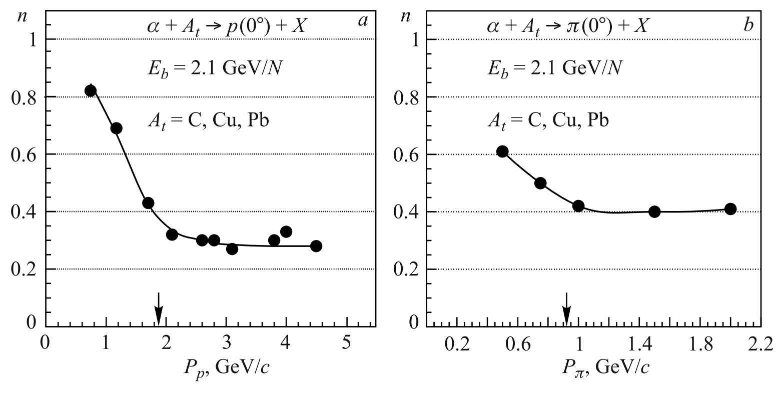

Fig. 1. Behavior of the exponent of the power approximation (see (5)) of the inclusive cross section for the fragmentation of incident alpha particles into protons (a) and pions (b) emitted at zero angle. The arrow indicates the beginning of the cumulative area. The solid line is drawn by eye. According to the data of [6] and [7]

atomic mass of the target nucleus with the transition from a noncumulative to a cumulative area may serve as an additional argument in favor of the ranging the reactions with production of cumulative particles in a separate class of reactions. In the works [6] and [7], the data similar to this shown in Fig. 1 were obtained for $A_b = D, {}^4\mathrm{He}, \mathrm{C}$. The paper [12] presents the experimental results which confirm the peripherical character of dependence for medium and heavy nuclei for the fragmentation of deuteron into cumulative negative pions emitted at zero angle. According to the same paper, the dependence for the fragmentation on the targets of light nuclei is stronger, $n > 1/3$, as seen in Fig. 2. Interestingly, the medium and heavy nucleus ($A \geqslant 12$) cross section dependence on the atomic mass of the fragmenting nucleus looks much more stronger $d\sigma \sim A^\alpha$ with $\alpha \approx 1$ for pions and $\alpha \approx 4/3$ for protons (see, for example [2, 4, 5, 13–15]). This serves to further illustrate the fact that the colliding nuclei are included asymmetrically in the reaction with production of cumulative particles. The dependence of the cross sections of cumulative particles on the atomic mass of the fragmenting nucleus is influenced by both the cumulative particle scattering on nucleons of the fragmenting nucleus (see [16]) and the mechanism of production of cumulative particle (see, for example, [2]). In this paper, based on the modeling of the nucleon–nucleon scattering in the colliding nuclei,we analyzed the dependence of the cross section of the incident deuteron fragmentation into cumulative pions and protons on the atomic mass of the target nucleus, that is, the reaction

$$D + A_t \to (\pi(0^\circ), p(0^\circ)) + X. \tag{6}$$

We have shown that taking into account the scattering of cumulative particles explains the experimental data. This can significantly narrow the class of models

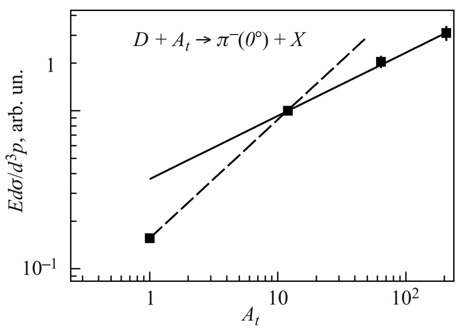

Fig. 2. Experimental data on the invariant cross sections of the cumulative π^- production, emitted at zero angle, on the atomic weight of the target nuclei in the fragmentation of incident deuterons ($P_\pi = 3$ GeV$/c$, $P_D = 7.3$ GeV$/c$). The solid line corresponds to the dependence $d\sigma \sim A_t^{0.4}$; the dashed one, to $d\sigma \sim A_t^{0.75}$. According to the data of [12]

proposed to explain the production of cumulative particles and to obtain the space–time picture of cumulative particle production in nucleus–nucleus interactions.

SIMULATION PROCEDURE

The interaction of the colliding nuclei and produced protons and pions is described on the basis of nucleon–nucleon scattering. Let us dwell on the details of modeling one event. At the first stage, the configuration of the colliding nuclei is determined, i. e., the position of nucleons is calculated. For medium and heavy nuclei ($A_t \geqslant 12$), the probability of finding a nucleon in a nucleus can be obtained from the distribution of Woods–Saxon [17]:

$$P_A(r) = \frac{N}{1 + \exp{(r - R_A)/d)}}, \tag{7}$$

where $d = 0.54$ fm is the diffuseness parameter;

$$R_A = 1.16(1 - 1.16A^{-1/3})A^{1/3}, \tag{8}$$

where R_A is the nucleus radius, and N is the normalization constant, which is chosen from the standard normalization condition

$$\int\limits_0^\infty P_A(r) r^2\, dr = 1. \tag{9}$$

For the deuteron, the distribution was specified in accordance with the Hyulten wave function [18]:

$$P_D(r_{pn}) = \frac{ab(a+b)}{2(b-a)^2} \left(\frac{e^{-2ar_{pn}} + e^{-2br_{pn}} - 2e^{-(a+b)r_{pn}}}{r_{pn}^2} \right), \tag{10}$$

where r_{pn} is the distance between the proton and the neutron; $a = 0.228$ fm^{-1}, and $b = 1.18$ fm^{-1}. For ^{4}He the density is chosen as a Gaussian distribution [19]:

$$P_\alpha(r) = \frac{4}{\sqrt{\pi} d^3} \exp{(-r^2/d^2)}, \tag{11}$$

and $d = 1.7$ fm. The center of the target nucleus was chosen at the origin. The positions of all the nucleons of the target nucleus were calculated using one of the distributions (7), (10), and (11), and the direction of the radius vector played out evenly over the full solid angle. Position of the center of the incident nucleus on the plane perpendicular to the Z-collisions axis played out uniformly inside the circle of radius R_{xy}:

$$R_{xy} = \sqrt{x^2 + y^2} \leqslant \beta(R_{A_b} + R_{A_t} + r_{NN}), \tag{12}$$

here R_{A_b} and R_{A_t} are «radii» of the beam and target nuclei, respectively. For medium and heavy nuclei, the radius is calculated from (8), for the deuteron $R_D = 5$ fm is taken, and for alpha particles $R_\alpha = 3$ fm. The nucleon interaction radius r_{NN} for the discussed energy region is chosen to be equal to $r_{NN} = 1.12$ fm. This value is discussed in detail below. It was found that for $\beta \geqslant 1.2$, the results of the calculation of total cross sections of interaction of colliding nuclei cease to depend on this value with a precision better than 0.5%.

After fixing the coordinates of the nucleons in the colliding nuclei, the scattering of nucleons was simulated in the following way. Along the trajectory of the nucleon we select a cylinder of radius $r_{NN} = \sqrt{\sigma_{NN}/\pi}$. If such a cylinder gets a nucleon of the target nucleus, it is considered that this nucleon experiences a collision. Upon further consideration, the first in time collision is selected. Similarly the interaction of secondary hadrons is considered, but the velocity of the secondary hadron is no longer directed along the axis of the collision, and «radius» of the interaction depends on the scattering cross section of the secondary hadron, so for pions $r_{\pi N} = \sqrt{\sigma_{\pi N}/\pi}$.

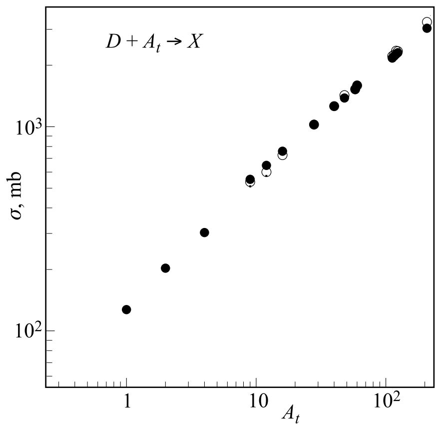

Fig. 3. The dependence of the reaction cross section (σ) of the deuteron on various targets at a kinetic energy $T_D = 97$ MeV. The filled circles are the simulation results, the open circles are the experimental data from work [27]

To evaluate the quality of modeling, we calculated the inelastic cross sections of the deuteron scattering with a wide range of target nuclei. From Fig. 3, one can see that the experimental data for the inelastic cross sections is in good agreement with our simulation results. In further calculations we used the following values of the total cross sections of the nucleon–nucleon scattering

$$\sigma_{NN} = 45 \text{ mb} \implies r_{NN} = 1.197 \text{ fm}. \qquad (13)$$

And for the pion–nucleon scattering:

$$\sigma_{\pi N} = 30 \text{ mb} \implies r_{\pi N} = 0.977 \text{ fm}, \tag{14}$$

The coordinates of cumulative particles were chosen equal to coordinates of a nucleon from the deuteron, that firsty was scattered on a nucleon of the target nucleus.

THE SIMULATION RESULTS

We begin with a discussion of a reaction of fragmentation of a deuteron beam into cumulative protons:

$$D + A_t \to p(0^\circ) + X. \tag{15}$$

In the simulation of cross sections for this reaction two mechanisms have been considered. In the first mechanism, cumulative protons are produced just after the first collision of the deuteron nucleons with a nucleon from the target nucleus. This mechanism is called «direct». The second of the possible mechanisms of the cumulative proton production in the fragmentation of the deuterium nucleus is called «cascade» mechanism. In this mechanism, cumulative protons are produced after two collisions with the nucleons of the target nucleus. In the first collision of a deuteron with a nucleon of the target nucleus, a proton is produced (not necessarily at a zero angle), which results in the production of cumulative protons (at zero angle) in the next collision. Let us consider each of these mechanisms.

1. In the «direct» mechanism, it is assumed that the detected proton is produced in the first collision of a deuteron with a nucleon of the target nucleus, and leaves the nucleus of the target without collisions. This means that the cross section is defined by the following expression:

$$\sigma(p) = \sigma(D + N \to p + X) \int_0^\infty db\, b \int_{-\infty}^\infty dz\, n(\rho)\; W_d(b,z)\; \bar{W}_p(b,z), \tag{16}$$

where $W_d(b,z)$ is the probability of deuteron with an impact parameter b to pass without collision to a point at z (the center of the nucleus of the target is at beginning of the axes, and the axis z is directed along the beam), and $\bar{W}_p(b,z)$ is the probability of born proton to leave the nucleus of the target without collisions. Obviously, with such a mechanism, the largest contribution to the cross section $\sigma(p)$ is given by the impact parameters, for which the total path traveled by the deuteron and proton is the sum of the mean free path of the proton and deuteron $l = \lambda_d + \lambda_p$. For medium and heavy nuclei, $\lambda_p \approx 1.6$ fm and $\lambda_d \approx 1$ fm. This

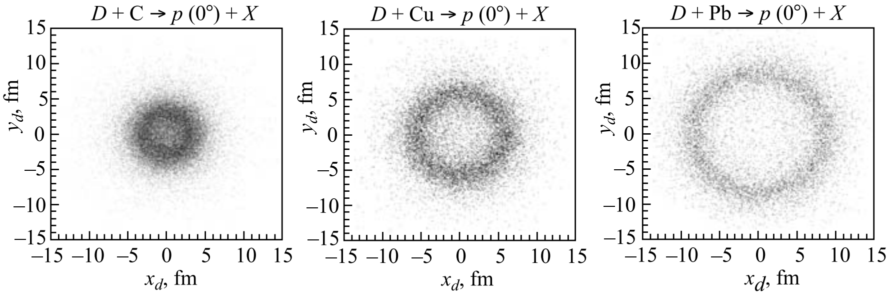

Fig. 4. Distribution of points of cumulative protons production in the plane perpendicular to the axis of the collision, as per the model of the direct mechanism, from left to right for the nuclei of C, Cu, and Pb, respectively. In all cases, the nucleus center is at the origin

means that for a nucleus with radius R_A the greatest contribution is coming from impact parameters equal to $b_p = \sqrt{R_A^2 - l^2/4} = \sqrt{R_A^2 - 1.3^2}$. These estimates give: for copper $R_{\mathrm{Cu}} = 4.8$ fm and $b_p = 4.6$ fm, and for lead $R_{\mathrm{Pb}} = 7$ fm and $b_p = 6.8$ fm. Thus, on a qualitative level it is clear why the «direct» mechanism of cumulative protons production leads to the «peripheral» nature of the dependence of the cross section on the atomic mass of the target nucleus. The simulation results for the nuclei Pb, Cu, and C (the nuclei that were used in the work [7]), presented in Fig. 4, confirm that for heavy and medium nuclei the main contribution to the cross section of cumulative proton production is coming from the large (with respect to the radius of the nucleus) impact parameters. A simulation study was carried out in which the effective length of cumulative protons formation was taken into account. The simplest way to take into account the formation length l_f was reduced to the assumption that after its production cumulative proton does not interact until carries the distance l_f. In Table 1, simulation results are presented for different values of formation lengths and the target nuclei of Pb, Cu, and C, used in [7]. The simulation results were approximated by a power law. The parameters were found by minimizing the standard expression for χ^2 for Pb, Cu, and C:

$$\chi^2 = \sum_{i=1}^{3} \frac{(d\sigma(A_i) - C \cdot A_i^n)^2}{(\Delta\sigma(A_i))^2} \quad , \tag{17}$$

The resulting values of the exponent n and χ^2 calculated from power approximation are presented in Table 1. Here the errors of cross sections were taken equal to 2%. Since the absolute value of the cross sections was not calculated, we present data normalized to the cross section for the carbon nucleus. From Table 1, one can see that «direct mechanism» with a zero formation length gives

Table 1. Invariant cross section of cumulative protons ($d\sigma(D + A_t \to p(0^\circ))$) for $A_t = \mathrm{C, Cu, Pb}$ and for different proton formation length. The cross section is expressed in arbitrary units

l_f, fm	$Ed\sigma(\mathrm{C})/d^3p$	$Ed\sigma(\mathrm{Cu})/d^3p$	$Ed\sigma(\mathrm{Pb})/d^3p$	$n \pm \Delta n$	χ^2
0	1.	1.92	2.8	0.36 ± 0.02	4
1	1.09	2.13	3.1	0.33 ± 0.05	6
2	1.19	2.37	3.41	0.37 ± 0.03	8
3	1.28	2.63	3.78	0.38 ± 0.03	11
4	1.35	2.90	4.17	0.40 ± 0.04	16
5	1.4	3.14	4.58	0.42 ± 0.05	21
6	1.42	3.37	5.00	0.45 ± 0.05	26
7	1.42	3.57	4.51	0.47 ± 0.05	29

results close to the experimental data with $n = 0.36 \pm 0.02$ and acceptable value of $\chi^2 = 4$.

2. Another mechanism, which contributes to the production of cumulative protons is a mechanism in which the proton is scattered on another nucleon of the nucleus of the target and is detected after that. Here is a view of cascade passing the process according to the following scheme:

$$\begin{array}{lll} D + N_1 \to & p_{\mathrm{cs}} & +X_1 \\ & \downarrow & \\ & p_{\mathrm{cs}} & +N_2 \to p(0^\circ) + X_2. \end{array} \quad (18)$$

For such a mechanism one should take into account that the intermediate proton p_{cs} has a momentum greater than the momentum of the proton detected. This means that the intermediate proton is also cumulative. To calculate the contribution of the cascade process (18) we need to specify both the cross section of the intermediate proton p_{cs} in the reaction $D + N_1 \to p_{\mathrm{cs}} + X_1$ and the scattering cross section for the second proton–nucleon reaction in the cascade process $p_{\mathrm{cs}} + N_2 \to p(0^\circ) + X_2$. Let us start with the cross section of production for cumulative protons. In the experiment it was established that, apart from independence of the collision energy, the spectra of cumulative particles obey a number of scaling relations. In particular, to asess the contribution of the cascade process, we will use the fact that the shape of the spectra of cumulative particles submitted according to the scale of cumulative variable does not depend on the type of a detected (cumulative) particle (see, [4]). The cumulative scaling variable (the cumulative number of X_c), gives an idea of how far below the threshold a cumulative particle is (see [4], [20]). This variable is defined as the minimal part of 4-momentum per nucleon fragmenting nucleus, necessary in order to the product inclusive hadron be kinematically permitted. In the reference frame in which the fragmenting nucleus is at rest, the cumulative number is equal to the minimal target mass (in units of the nucleon mass) for which the production of corresponding cumulative particles is permitted, that is, for cumulative particles of $X_c > 1$. In the relativistically invariant form, the cumulative number is given by the following expression (see, [20]):

$$X_c = \frac{(P_{\rm II} P_c) + m_N m_2 + (m_2^2 - m_c^2)/2}{(P_{\rm I} P_{\rm II}) - m_N^2 - (P_{\rm I} P_c) - m_N m_2}, \tag{19}$$

here $P_{\rm I}, P_{\rm II}$ — four-momentum per nucleon for the fragmenting particle and the particle which is not fragmenting, respectively; P_c — four-momentum of the cumulative particle; m_N — nucleon mass; m_2 — mass of the lightest particle, which has to be produced due to the conservation laws in the reaction (i.e., in addition to the conservation of 4-momentum, i.e., conservation of strangeness, baryonic number, etc.), in particular, for the production of cumulative protons $m_2 = -m_N$. From (19) it follows that for the fragmentation of a beam deuteron into cumulative protons, the cumulative number is equal to:

$$X_c = \frac{E_b m_N - m_N^2}{E_b m_N - 2m_N^2 - E_b E_p + p_b p_p \cos\Theta_p}, \tag{20}$$

where $E_p,\ p_p,\ \Theta_p$ are energy, momentum and emission angle of the cumulative proton, and $E_b,\ p_p$ are the energy and momentum per nucleon for a beam. It was established experimentally (see [20, 21]), that cross section of the cumulative proton production can be approximated by the exponential dependence on the cumulative scaled variable X_c (cumulative number)

$$E\frac{d\sigma}{d\mathbf{p}} = B \exp\left(-X_c/\langle X_c\rangle\right). \tag{21}$$

For the fragmentation of a deuteron into cumulative protons, the inverse slope parameter is equal to $\langle X_c\rangle = 0.12$. For the cross section of the intermediate proton scattering on the target nucleon, we used the approximation which is often used in the description of the elastic scattering (see, [22, 23]):

$$E\frac{d\sigma}{d\mathbf{p}} = C \cdot \exp\left(B_0 \cdot t\right), \tag{22}$$

where t is the four-momentum squared equal to:

$$t = (P_{\rm cs} - P(0^\circ))^2, \tag{23}$$

and $P_{\rm cs}$ and $P(0^\circ)$ are the four-momenta of the intermediate and detected protons. Factor C was calculated from the condition of equality of the integral of differential cross section (22) to the total cross section of the proton–nucleon scattering at energies of a few GeV:

$$\sigma_{\rm tot}(NN) = 45{\rm mb} = \int \left(E \frac{d\sigma}{d\mathbf{p}} \right) \frac{d\vec{\mathbf{p}}}{E}. \qquad (24)$$

The simulations were carried out for values of slope parameter of $B_0 = 7$ GeV^{-2}, $B_0 = 10$ GeV^{-2}, $B_0 = 14$ GeV^{-2}, and $B_0 = 0$ GeV^{-2}. The calculation results for the cross sections of the cascade process for the considered types of sections are shown in Table 2. From this table one can see that the largest cross section of the cascade process was obtained for the sharpest decline ($B_0 = 14$ GeV^{-2}) of the cross section of rescattering on the momentum transfer. But even in this

Table 2. Invariant cross sections obtained from the simulation of the process of fragmentation of the incident deuterium into cumulative protons for different nuclei targets and different slope parameters B_0 in the approximation of the nucleon–nucleon cross section by the expression (22). The cross section is expressed in arbitrary units

Kind of reaction	B_0	$E\frac{d\sigma({\rm H})}{d^3p}$	$E\frac{d\sigma({\rm D})}{d^3p}$	$E\frac{d\sigma(^4{\rm He})}{d^3p}$	$E\frac{d\sigma({\rm C})}{d^3p}$	$E\frac{d\sigma({\rm Cu})}{d^3p}$	$E\frac{d\sigma({\rm Pb})}{d^3p}$
Direct production		0.227	0.382	0.527	1	1.926	2.799
Cascade	14.		$1.27\cdot10^{-3}$	$1.81\cdot10^{-3}$	$4.42\cdot10^{-3}$	$1.15\cdot10^{-2}$	$1.83\cdot10^{-2}$
Cascade	10.		$0.98\cdot10^{-3}$	$1.4\cdot10^{-3}$	$3.4\cdot10^{-3}$	$0.89\cdot10^{-2}$	$1.41\cdot10^{-2}$
Cascade	6.		$0.71\cdot10^{-3}$	$1.01\cdot10^{-3}$	$2.48\cdot10^{-3}$	$0.64\cdot10^{-2}$	$1.02\cdot10^{-2}$
Cascade	0.		$3.62\cdot10^{-5}$	$5.17\cdot10^{-5}$	$1.26\cdot10^{-5}$	$3.28\cdot10^{-4}$	$5.22\cdot10^{-4}$

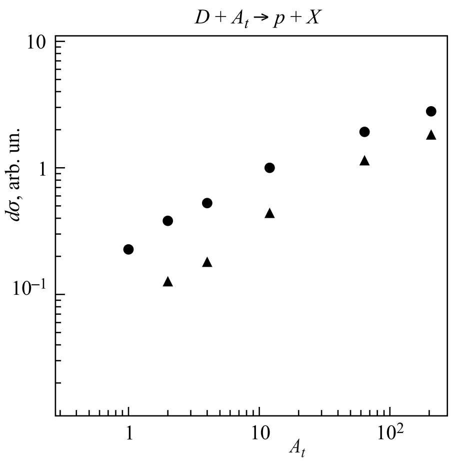

Fig. 5. Invariant cross section ($d\sigma = E d\sigma/d^3p$) for the deuteron fragmentation into cumulative protons from the atomic mass for various processes. The circles is the «direct process» (see (16)), the triangles is the cascade process cross section multiplied by 100 (see (18)) for the values of the slope parameter $B_0 =$ 14 GeV^{-2} (see (22))

case, the cross section of the cumulative proton production in cascade process does not exceed 1% in comparison to the direct process. Besides, the cross section of cascade process, as one can see in Fig. 5, has a steeper dependence on the atomic mass of the target nucleus. So, for medium and heavy target nuclei (C, Cu, Pb) the cascade mechanism leads to an exponent $n = 0.5$ for a power approximation of the cross section (see (22)). From these estimations one can conclude that the contribution of the cascade mechanism into the fragmentation cross section of the deuteron into cumulative protons is small. The cascade process, in which a cumulative proton is produced through an intermediate pion, is prohibited because of the energy conservation for the beam energy of about 10 GeV/N.

To compare the results of modeling the dependence on the atomic weight of the nuclei on which the fragmentation occurs, the experimental data from [24] was used. In work [24], the experimental data for the production of cumulative particles (in particular, protons and pions) is presented for the following reactions:

$$B(p = 4.5\ \text{GeV}/(c \cdot n) + A_t \to c\,(0.5\ \text{GeV}/c, 120^\circ) + X. \qquad (25)$$

The fact that the cumulative particle is detected in the backward hemisphere means that the cumulative particles are fragments of the target nuclei. The B nucleus, on which the fragmentation occurred, was a beam particle. To make a comparison with the production at the incident particle, go to the anti lab frame. For a carbon target it gives:

$$C(p = 4.5\ \text{GeV}/(c \cdot n)) + A_t(p, D, \alpha, C) \to p\,(7.2\ \text{GeV}/c,\ 2^\circ) + X, \qquad (26)$$

and the corresponding data is shown in Table 3. Errors in the experimental data are determined mainly not by statistical accuracy but by accuracy of the normalization of beam intensities. Absolute normalization was performed using the method of induced activity and was 10% for the proton beam, and 15% for the other of the used beams. The results of simulation and the experimental data for the reaction (24) are presented in Fig. 6.

Table 3. Invariant cross section of cumulative protons ($X_c = 1.55$) production for reactions (26) from the data of [24]. The cross section is expressed in arbitrary units

A_b, fm	$Ed\sigma(\text{H})/d^3p$	$Ed\sigma(\text{D})/d^3p$	$Ed\sigma(^4\text{He})/d^3p$	$Ed\sigma(\text{C})/d^3p$
C	0.30 ± 0.05	0.49 ± 0.10	0.51 ± 0.10	1.0

Let us turn to the reaction of deuteron fragmentation into cumulative pions:

$$D + A_t \to \pi(0^\circ) + X. \qquad (27)$$

In this case, along with the direct mechanism of cumulative pions production there are two types of cascade processes:

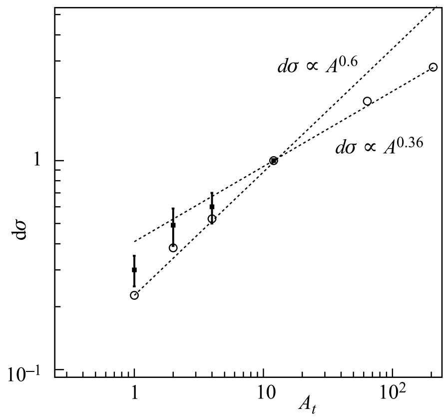

Fig. 6. Invariant cross section for the deuteron fragmentation ($d\sigma = Ed\sigma/d^3p$) into cumulative protons, emitted at zero angle, for the reaction $D + A_t \to p(0^\circ) + X$). The open circles are taken from simulation. The closed rectangles are experimental data for the reaction (24) from [24]. The cross section is expressed in arbitrary units

1) cascade process with an intermediate pion:

$$\begin{array}{llll} D + N_1 \to & \pi_{\rm cs} & +X_1 & \\ & \downarrow & & \\ & \pi_{\rm cs} & +N_2 \to & \pi(0^\circ) + X_2, \end{array} \tag{28}$$

2) cascade process with an intermediate proton (nucleon):

$$\begin{array}{llll} D + N_1 \to & p_{\rm cs} & +X_1 & \\ & \downarrow & & \\ & p_{\rm cs} & +N_2 \to & \pi(0^\circ) + X_2. \end{array} \tag{29}$$

The experimental values of the reaction cross sections (25), obtained in [12], as well as the data obtained from the simulation for different lengths of pions formation are shown in Table 4 with the corresponding value of χ^2 also presented.

It should be noted that for the pions in the antilaboratory frame the reaction of (25) studied in [24] is

$$C(p = 4.5\ \mathrm{GeV}/(c \cdot n)) + A_t(p, D, \alpha, C) \to \pi^+(4.6\ \mathrm{GeV}/c,\ 3^\circ) + X, \tag{30}$$

The cross sections for this reaction taken into account with the uncertainty in the absolute normalization are shown in Table 5. The experimental data and simulation results for several formation lengths are shown in Fig. 7. From this figure it follows that the available experimental data of deuteron fragmentation into cumulative pions is in a good agreement with the theoretical calculations for the formation length $1\ \mathrm{fm} \leqslant l_{\rm f} \leqslant 4\ \mathrm{fm}$. Accuracy of the experimental data [12] does not allow for more precise conclusions about the magnitude of the formation length of pions. Moreover, as can be seen in Fig. 7, the data of the fragmentation

Table 4. Invariant cross sections of the production of cumulative protons obtained in the work [12], calculated in the model under discussion for a variety of pion formation lengths $d\sigma(D + A_t \to \pi(0^\circ)$, and $A_t = \mathrm{H, C, Cu, Pb}$. The cross section is expressed in arbitrary units and was normalized on the cross section of a carbon target for both experimental and simulated data (for simulations we used the cross section of a carbon target for $l_f = 0$)

l_f, fm	$d\sigma(\mathrm{H})/d^3p$	$Ed\sigma(\mathrm{C})/d^3p$	$Ed\sigma(\mathrm{Cu})/d^3p$	$Ed\sigma(\mathrm{Pb})/d^3p$	χ^2
exp. [12]	0.16 ± 0.02	1.0 ± 0.1	2.0 ± 0.2	3.1 ± 0.3	
0	0.21	1.0	2.03	3.05	2.6
1	0.21	1.07	2.24	3.31	1.6
2	0.21	1.14	2.42	3.61	0.7
3	0.21	1.20	2.52	3.93	0.4
4	0.21	1.24	2.82	4.26	0.7
5	0.21	1.27	3.01	4.62	1.4
6	0.21	1.30	3.19	5.00	2.5
7	0.21	1.30	3.36	5.31	4.3

Table 5. Invariant cross section of the production of cumulative protons ($X_c = 1.2$) for reactions (30) from the data of [24]. The cross section is expressed in arbitrary units

A_b, fm	$Ed\sigma(\mathrm{H})/d^3p$	$Ed\sigma(\mathrm{D})/d^3p$	$Ed\sigma(^4\mathrm{He})/d^3p$	$Ed\sigma(\mathrm{C})/d^3p$
C	0.20 ± 0.04	0.38 ± 0.08	0.51 ± 0.11	1.0

of the carbon on light nuclei (see (26)) of [24] are also in a good agreement with the simulation results for the above-mentioned lengths of formation of pions. In the calculations of the contribution of the cascade process with the intermediate pion (28) we used the same approximation for the scattering cross section of the intermediate pion–nucleon of the target nucleus as for the scattering of the intermediate proton in the cascade of cumulative protons (22). The only difference is in the value of the normalization factor from the normalization condition (24) $\sigma_{\pi N}$= 30 mb. Since the intermediate pion has momentum greater than the detectable one, the production process of this pion was calculated in accordance with the universal dependence of cross section of the cumulative variable (see (21)). The results of the calculation of the cross section of the cumulative pion production in the cascade process with the intermediate pion (28) for different values of the slope parameter A_t are presented in Table 6. From this table it follows that:

• the cascade process cross section has more sharper, than in the experiment,dependence on the atomic mass of the target nucleus;

• the cross section of the cascade process with the intermediate pion does not exceed 0.6% of the cross section of the direct mechanism of cumulative pions production.

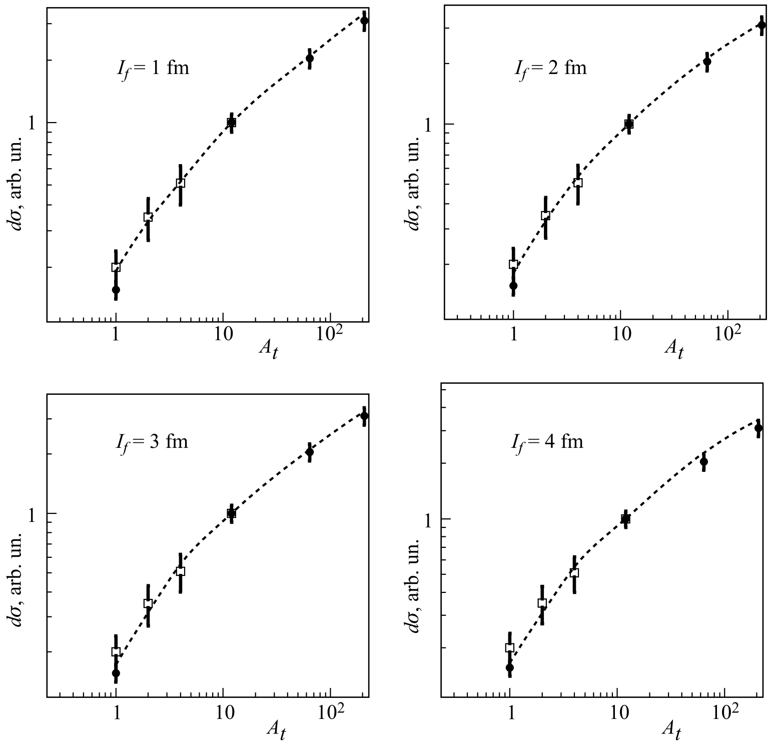

Fig. 7. Invariant cross section for the deuteron fragmentation ($d\sigma = Ed\sigma/d^3p$) into cumulative protons, emitted at zero angle, for the reaction $D + A_t \rightarrow p(0°) + X$. The open circles are taken from simulation. The closed rectangles are experimental data for the reaction (24) from [24]. The cross section is expressed in arbitrary units

Calculating a contribution of the cascade process with the intermediate proton (29), we took into account that a cumulative pion can be born in a collision with a nucleon target only if the intermediate proton is also cumulative. Therefore, for the production of the intermediate proton we used the approximation of the cross section in dependence on the scale of the cumulative variable (see (21)). For the cross section of the pion production by the intermediate proton, the approximation proposed in [25] (equation (4) with the parameters from Table 5) was used. Calculations have shown that the contribution of the considered cascade process (29) does not exceed 10^{-5} in comparison to the direct mechanism, even for the fragmentation on the lead nucleus. Thus, as in the case of protons, cascade mechanisms for pion production are negligible.

One should note that in all our calculations we have overestimated the cross section of the cascade mechanism (even for the elastic scattering, see for exam-

Table 6. Invariant cross sections obtained from the simulation of the process of fragmentation of the incident deuterium into cumulative protons for the cascade process with a pion in the intermediate state (28) for a variety of target nuclei and different slope parameters B_0 in the approximation of the nucleon–nucleon cross section by the expression (22). The cross section is expressed in arbitrary units

Kind of reaction	B_0	$E\frac{d\sigma(\mathrm{H})}{d^3p}$	$E\frac{d\sigma(\mathrm{D})}{d^3p}$	$E\frac{d\sigma(^4\mathrm{He})}{d^3p}$	$E\frac{d\sigma(\mathrm{C})}{d^3p}$	$E\frac{d\sigma(\mathrm{Cu})}{d^3p}$	$E\frac{d\sigma(\mathrm{Pb})}{d^3p}$
Direct production		0.21	0.324	0.476	1	2.03	3.05
Cascade (28)	14.		$0.077 \cdot 10^{-2}$	$0.12 \cdot 10^{-2}$	$0.35 \cdot 10^{-2}$	$0.99 \cdot 10^{-2}$	$1.86 \cdot 10^{-2}$
Cascade (28)	10.		$0.071 \cdot 10^{-2}$	$0.11 \cdot 10^{-2}$	$0.32 \cdot 10^{-2}$	$0.91 \cdot 10^{-2}$	$1.7 \cdot 10^{-2}$
Cascade (28)	6.		$0.048 \cdot 10^{-2}$	$0.082 \cdot 10^{-2}$	$0.23 \cdot 10^{-2}$	$0.67 \cdot 10^{-2}$	$1.26 \cdot 10^{-2}$
Cascade (28)	0.		$0.26 \cdot 10^{-4}$	$0.46 \cdot 10^{-4}$	$1.18 \cdot 10^{-4}$	$3.35 \cdot 10^{-4}$	$6.3 \cdot 10^{-4}$

ple [22, 26]). Hence we can deem that the fragmentation of the deuterium into the cumulative pions cross section is described by the «direct» mechanism in which the cumulative pions are produced in the first deuteron collision with a nucleon of the target nucleus.

CONCLUSION

In conclusion, we list the main results of the article.

• The reaction of the fragmentation of the incident deuterons into cumulative pions and protons on targets with different atomic mass was studied. The simulation of the dependence of the cross section on atomic mass of the target, based on the description of deuteron fragmentation reaction, was performed based on the nucleon–nucleon scattering.

• Based on this approach, it was shown that the main contribution to the cross section gives the particle production in the «direct» process, i. e., the process in which the cumulative particle (proton or pion) is produced in the deuteron collision with a nucleon of the target nucleus.

• It is shown that the contribution of «cascading» processes in which the cumulative particle is created after the production at the intermediate particle (proton or pion), is less than one per cent in relation to the «direct» process.

• The proposed approach describes experimental data depending on the atomic mass of the target of the cross section for the reaction of deuteron fragmentation into cumulative pions and protons without additional parameters.

The question dependence of the cross section of the incident nucleus fragmentation into cumulative particles on the atomic mass of the target nucleus,when the mass of the nucleus of the beam is greater than or equal to the mass of the target nucleus, remains open. The results of simulations, carried out for the situation when the fragmenting nucleus is smaller (by number of nucleons) than target nuclei, allow us to hope that the proposed approach is adequate for any relation between the atomic masses of the colliding nuclei.

REFERENCES

1. *Baldin A. M.* // Physics of Elementary Particles and Atomic Nuclei. 1977. V. 8(3). P. 429.
2. *Efremov A. V.* // Physics of Elementary Particles and Atomic Nuclei. 1982. V. 13(3). P. 613.
3. *Frankfurt L. L, Strikman M. I.* // Phys. Rep. 1981. V. 76. P. 215.
4. *Stavinski V. S.* // Physics of Elementary Particles and Atomic Nuclei. 1979. V. 10(5). P. 949.
5. *Lukyanov V. K., Titov A. I.* // Physics of Elementary Particles and Atomic Nuclei. 1979. V. 10(4). P. 815.
6. *Anderson L. et al.* // Phys. Rev. C. 1983. V. 28(3). P. 1224.
7. *Moeller E. et al.* // Phys. Rev. C. 1983. V. 28(3). P. 1246.
8. *Abraamyan Kh. et al.* // Phys. Lett. B. 1994. V. 323. P. 1.
9. *Afanasiev S. V. et al.* // Physica Scripta. 1993. V. 48. P. 124.
10. *Afanasiev S. V. et al.* // JINR Rapid Communications. 1993. 1[58]–93. P. 21.
11. *Afanasiev S. V. et al.* // JINR Rapid Communications. 1993. 3[60]–93. P. 40.
12. *Anisimov Yu. S. et al.* // Yadernaya Fizika (Rus.). 1997. V. 60(6). P. 1070.
13. *Belyaev I. M. et al.* // Yadernaya Fizika (Rus.). 1993. V. 56(10). P. 135.
14. *Belyaev I. M. et al.* // JINR Rapid Communications. 1989. 1[34]–89. P. 5.
15. *Bondarev V. K.* // Physics of Elementary Particles and Atomic Nuclei. 1997. V. 28(1). P. 13.
16. *Zolin L. S., Peresedov V. F.* // JINR Rapid Communications. 1992. 3[54]. P. 593.
17. *Amaya A.,Chacon E.* // Comp. Phys. Comm. 1992. V. 71. P. 159–172
18. *Sagavara M., Hulthen L.* // Handb. Phys. 1957. V. 39. P. 1.
19. *Barlet R. C., Jakson D. F.* // Nuclea Sizes and Structure. N. Y.: Oxford Univ. Press. 1997.
20. *Litvinenko A. G., Malakhov A. I., Zarubin P. I.* // JINR Rapid Communications. 1993. 1[58]–93. P. 27.
21. *Baldin A. M. et al.* JINR Commun. E1-82-472. Dubna, 1982.
22. *Okorokov V. A.* arXiv:0811.0895v2 [hep-ph]. 2008.
23. *Okorokov V. A.* arXiv:0907.0951v2 [hep-ph]. (2009).
24. *Bondarev V. K. et al.* JINR Commun. E1-93-84. Dubna, 1993.
25. *Barashenkov V. S., Slavin N. V.* // Physics of Elementary Particles and Atomic Nuclei. 1984. V. 15(5). P. 997.
26. *Waz Z.* // Acta Physica Polonica. B. 1981. V. 12. P. 967.
27. *Auce A. et al.* // Phys. Rev. C. 1996. V. 53. P. 2919.

Received on November 6, 2013.

Редактор *Э. В. Ивашкевич*

Подписано в печать 27.12.2013.
Формат $60 \times 90/16$. Бумага офсетная. Печать офсетная.
Усл. печ. л. 1,18. Уч.-изд. л. 1,59. Тираж 285 экз. Заказ № 58153.

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