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**THE EMC RATIO AS A FUNCTION  
OF  $x, Q^2$  IN THE RESCALING MODEL**

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Recent discovery of the EMC-effect has shown an essential difference between the nucleus and nucleon structure functions (SF). To explain the EMC-effect, different schemes have been suggested<sup>/2-5/</sup>. In the note we will follow the rescaling model<sup>/4,5/</sup>; the SF of the nuclei A and A' are connected by the relation

$$\frac{1}{A'} F_K^{A'}(x, Q^2) = \frac{1}{A} F_K^A(x, \bar{x}_{A'}(Q^2) \cdot Q^2).$$

We will get the parametrization for the ratio

$$R_{K,A}^{A'} = \frac{A F_K^{A'}(x, Q^2)}{A' F_K^A(x, Q^2)}.$$

Hereafter the index K takes values 2 and 3.

We use the parametrization obtained by Lopeo and Yndurain<sup>/6,7/</sup>

for the nucleon SF by analysing its behaviour for  $x \approx 0$  and  $x \approx 1$ .

The rescaling idea: the confinement radius  $\lambda_A$  depends on the nucleus type (that is, on A). Hence there appears the A-dependence of scale  $\mu_A^2$  of the  $Q^2$ -evolution of the SF moments<sup>/4,5/</sup>:

$$-\bar{x}(\bar{x}_{A'}(Q^2) \cdot Q^2) = \frac{\bar{x}(\mu_A^2)}{\bar{x}(\mu_{A'}^2)} \bar{x}(Q^2),$$

where

$$\bar{x} = \frac{g^2}{16\pi^2} \quad \text{and} \quad \frac{\mu_{A'}^2}{\mu_A^2} = \frac{\lambda_A^2}{\lambda_{A'}^2}.$$

The ratios of the confinement radii are given in paper<sup>/5/</sup>.

1. The regions  $x \rightarrow 1$ . The SF are in detail given in paper<sup>/6,7/</sup>. We consider their ratio  $R_{K,A}^{A'}$ . We get

$$R_{K,A}^{A'} \underset{x \rightarrow 1}{=} \left[ \frac{\bar{x}(\mu_{A'}^2)}{\bar{x}(\mu_A^2)} \right]^{-d_0} \frac{\Gamma(1+\nu(\bar{x})) (1-x)}{\Gamma(1+\nu(\bar{x}) - \frac{4C_F}{\beta_0} \ln \frac{\bar{x}(\mu_A^2)}{\bar{x}(\mu_{A'}^2)})}, \quad (1)$$

$$\text{where } \nu(\bar{x}) = \nu_0 - \frac{4C_F}{\beta_0} \ln \bar{x}(Q^2), \quad d_0 = \frac{4C_F}{\beta_0} \left( \frac{3}{4} - \gamma \right)$$

and  $\gamma$  is the Euler constant.

Since  $\left| 1 - \frac{\bar{x}(\mu_{A'}^2)}{\bar{x}(\mu_A^2)} \right| \ll 1$ , expression (1) can be rewritten in the form

$$R_{K,A}^{A'} = \left[ \frac{\bar{x}(\mu_{A'}^2)}{\bar{x}(\mu_A^2)} \right]^{d_x(\bar{x})}, \quad (2)$$

where

$$d_x(\bar{x}) = \frac{4C_F}{\beta_0} \left( \ln \frac{1}{1-x} + \Psi(1+\nu(\bar{x})) + \gamma - \frac{3}{4} \right).$$

2. The region  $x \rightarrow 0$ . The ratios of SF  $R_{K,A}^{A'}$  have the form<sup>/6,7/</sup>:

$$R_{3,A}^{A'} \underset{x \rightarrow 0}{=} \left[ \frac{\bar{x}(\mu_{A'}^2)}{\bar{x}(\mu_A^2)} \right]^{-d_{\lambda}^{NS}} \\ R_{2,A}^{A'} \underset{x \rightarrow 0}{=} \left[ \frac{\bar{x}(\mu_{A'}^2)}{\bar{x}(\mu_A^2)} \right]^{-d_{\lambda_S}^+} \left\{ 1 + \frac{A_{NS}}{A_S} (d_{\lambda_S}^+ - d_{\lambda}^{NS}) \ln \frac{\bar{x}(\mu_{A'}^2)}{\bar{x}(\mu_A^2)} \right\} \\ [d(Q^2)]^{(d_{\lambda_S}^+ - d_{\lambda}^{NS}) x^{\lambda_S - \lambda}} \approx \left[ \frac{\bar{x}(\mu_{A'}^2)}{\bar{x}(\mu_A^2)} \right]^{-d_{\lambda_S}^+}, \quad (3)$$

$$\text{where } d_{\lambda}^{NS} = -\frac{\gamma_{\lambda}^{NS}}{2\beta_0}, \quad d_{\lambda_S}^+ = -\frac{\gamma_{\lambda_S}^+}{2\beta_0}.$$

Expression (3) contains the anomalous dimensions  $\gamma_{\lambda}^{NS}$  and  $\gamma_{\lambda_S}^+$  obtained by analytical continuation from integer values of the argument  $n$  (as for the moments) to the region of noninteger values of  $\lambda$  and  $\lambda_S$  (see<sup>/7/</sup>). The values of  $\lambda, \lambda_S$  and  $\nu_0$  are fixed from the experimental data<sup>/8/</sup> and Regge theory<sup>/6/</sup>. We use  $\lambda \approx 0.5; \lambda_S = 1.6; \nu_0 = 0.47$ . Respectively, we get  $\gamma_{\lambda}^{NS} = -8.556$ ,  $\gamma_{\lambda_S}^+ = -9.718$ .

3. To find the  $R_{K,A}^{A'}$  parametrization in the whole region of  $x$ , we must connect the solutions (2) and (3). We do this as follows:

$$R_{3,A}^{A'} = \left[ \frac{\bar{\alpha}(\mu_A^2)}{\bar{\alpha}(\mu_{A'}^2)} \right]^{-d_{\lambda}^{NS}} (1-x^a) + \left[ \frac{\bar{\alpha}(\mu_A^2)}{\bar{\alpha}(\mu_{A'}^2)} \right] d_x(\bar{\alpha}) \cdot x^b \quad (4)$$

$$R_{2,A}^{A'} = \left[ \frac{\bar{\alpha}(\mu_A^2)}{\bar{\alpha}(\mu_{A'}^2)} \right]^{-d_{\lambda}^+} (1-x^a) + \left[ \frac{\bar{\alpha}(\mu_A^2)}{\bar{\alpha}(\mu_{A'}^2)} \right] d_x(\bar{\alpha}) \cdot x^b.$$

The parameters  $a$  and  $b$  are determined by the conditions:

1. The parametrization of a straight line must be a straight line. Hence,  $a=b$ .

2.  $R_{K,A}^{A'} = 1$  for  $x \approx 0.18$ . This conformity follows from the analysis of data /9-13/ and from the behaviour of the ratio calculated numerically /5/. Hence,  $a=0.5$ .

Using the relation for the coupling constants to the leading perturbation order:

$$\frac{\bar{\alpha}(\mu_A^2)}{\bar{\alpha}(\mu_{A'}^2)} = 1 + \beta_0 \bar{\alpha}(\mu_A^2) \ln \frac{\mu_{A'}^2}{\mu_A^2}$$

we arrive at the final expressions

$$R_{3,A}^{A'} = 1 - \bar{\alpha}(\mu_A^2) \ln \frac{\mu_A}{\mu_{A'}} \left[ \gamma_{\lambda}^{NS} (1-\sqrt{x}) + \gamma_x(\bar{\alpha}) \sqrt{x} \right] \quad (5a)$$

$$R_{2,A}^{A'} = 1 - \bar{\alpha}(\mu_A^2) \ln \frac{\mu_A}{\mu_{A'}} \left[ \gamma_{\lambda}^+ (1-\sqrt{x}) + \gamma_x(\bar{\alpha}) \sqrt{x} \right]. \quad (5b)$$

Let us compare the expression (5b) with the experimental data /1,9-12/ for  $A=Fe$  (Fig.1). The values  $\lambda_{Fe}/\lambda_1 = 1.153$  and  $\mu_1^2 = 0.5 \text{ GeV}^2$  are presented in paper /5/. We conclude that the SF of the nucleon and deuteron are the same /4,5/. The value of the QCD parameter is chosen to be  $\Lambda = 210 \text{ MeV}$  /14/. Note that the SLAC data /12/ are taken from the experiment on a copper target.

One can see that the parametrization (5b) agrees well with the data of different groups for  $0.2 \leq x \leq 0.7$ ; the data are obtained in large range of  $Q^2$ ; the  $Q^2$  dependence of parametrization (3) is extremely weak. It is twice logarithmic and it is an argument of the  $\Psi$ -function. This behaviour of the ratio  $R_{2,D}^{A'}$  has been observed in the experiment (see Fig.2 in paper /9/).

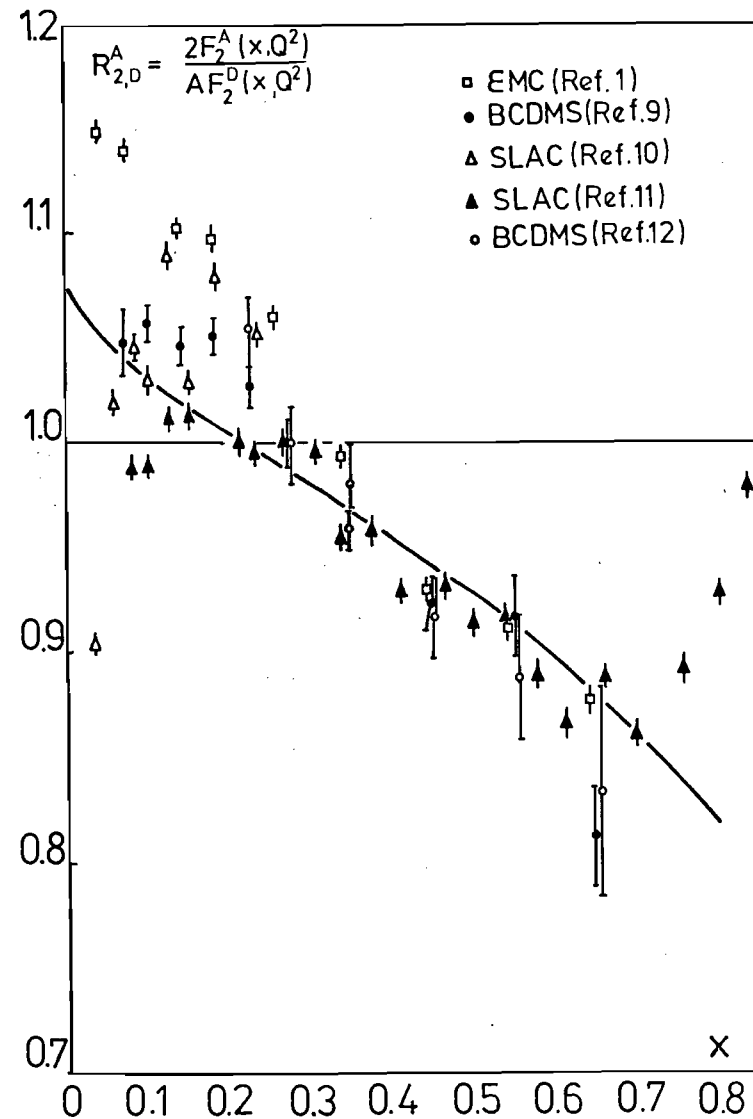


Fig.1. Comparison of the parametrization  $R_{2,D}^{Fe}$  with the experimental data of groups /1,9-12/. The data of groups /9,12/ (of groups /1,10,11/) are given only with statistical errors (without errors). The SLAC data /12/ are obtained on the copper target.

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Поведение EMC отношения как функции  $x, Q^2$   
в модели рескейлинга

Мы рассмотрели поведение отношения  $R_{K,A}^{A'} = \frac{AF_K^{A'}(x, Q^2)}{A'F_K^A(x, Q^2)}$

( $K=2,3$ ) при  $x \rightarrow 0,1$  в модели рескейлинга. Мы построили простую параметризацию для  $R_{K,A}^{A'}$ , совместную с поведением в точках  $x \approx 0,1$ . Параметризация крайне слабо зависит от  $Q^2$  и хорошо согласуется с экспериментальными данными в области  $0,2 \leq x \leq 0,7$ .

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The EMC Ratio as a Function of  $x, Q^2$   
in the Rescaling Model

We consider the ratio  $R_{K,A}^{A'} = \frac{AF_K^{A'}(x, Q^2)}{A'F_K^A(x, Q^2)}$  ( $K = 2,3$ )

as  $x \rightarrow 0,1$  in the rescaling model. We propose a simple parametrization for  $R_{K,A}^{A'}$  compatible with its behaviour at  $x \approx 0,1$ ; the parametrization slightly depends on  $Q^2$  and well agrees with experimental data in the region  $0.2 \leq x \leq 0.7$ .

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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